

Diagnostic Test

This first exercise will help you find out which areas of maths you need to work on before you start your A-Level Maths course. Do it before you work through the book — if you struggle with any of the questions, go straight to the relevant pages to brush up on your skills.

Once you've done all that, work through the rest of the book. You'll be able to recap and practise some useful GCSE topics and see how they'll lead into your A-Level work.

Types of Number and Fractions

These topics are covered in Section 1 — p.6-7.

1) Which of the following are integers?

4 -3.5 0.3 $\frac{4}{5}$ 8.99 -10 205 0

2) Which of the following values are rational, and which are irrational?

5.9 π $\sqrt{7}$ $\frac{1}{5}$ -6 $\sqrt{4}$ 13.978 2.1

3) Evaluate the following without using a calculator, giving your answers in their lowest terms. Give any answers larger than 1 as improper fractions.

a) $\frac{2}{9} \times \frac{3}{5}$ b) $\frac{1}{6} \div \frac{2}{3}$ c) $\frac{1}{12} + \frac{5}{6}$ d) $\frac{8}{5} - \frac{1}{7}$

Indices, Multiplying Out Brackets and Factorising

These topics are covered on p.8-11.

4) Simplify the following:

a) $x^7 \times x^2$ b) $10y^3 \div 5y$ c) m^0 d) $(2n^2)^5$

5) Write 5^{-2} as a fraction.

6) Evaluate the following without using a calculator:

a) $\left(\frac{3}{4}\right)^2$ b) $16^{\frac{1}{2}}$ c) $8^{\frac{2}{3}}$ d) $36^{-\frac{1}{2}}$

7) Multiply out the brackets and simplify your answers where possible.

a) $(x+4)(x-6)$ b) $(x+5)^2$ c) $(2x-1)(x+3)$ d) $(x+1)(x-4)(x+5)$

8) Factorise the following:

a) $5x+20$ b) $3a+12ab$ c) x^2-4 d) $9x^2-36$ e) x^2-5

Surds

This topic is covered on p.12-13.

9) Simplify the following:

a) $\sqrt{3} \times \sqrt{2}$ b) $(\sqrt{5})^2$ c) $\frac{\sqrt{30}}{\sqrt{6}}$ d) $\sqrt{12} + 2\sqrt{3}$ e) $(1 + \sqrt{7})^2$

10) Rationalise the denominators of the following:

a) $\frac{3}{\sqrt{2}}$ b) $\frac{\sqrt{5}}{2\sqrt{2}}$ c) $\frac{2}{3+\sqrt{6}}$ d) $\frac{\sqrt{2}}{1-\sqrt{5}}$

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Solving Equations and Rearranging Formulas

You'll find these on p.14-15.

11) Solve the following:

a) $5x - 2 = 8$ b) $3(x - 6) = 2(x - 4)$ c) $\frac{x+2}{3} + \frac{2x}{5} = x + 2$ d) $2x(x + 1) = 2x + 18$

12) Make x the subject of the following formulas:

a) $y = mx + c$ b) $y = \frac{3x+2}{5}$ c) $y = 2x^2z + 1$ d) $y = \frac{3x+1}{x-2}$

Quadratic Equations

Quadratics are covered
in Section 3 — p.16-21.

13) Solve the following by factorising:

a) $x^2 - 3x + 2 = 0$ b) $x^2 + 6x + 5 = 0$ c) $2x^2 - 3x - 5 = 0$ d) $3x^2 - 13x = -12$

14) Solve the following using the quadratic formula.
Give your answers to two decimal places.

a) $x^2 + 2x - 10 = 0$ b) $2x^2 - 5x - 1 = 0$

The formula is: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

15) Solve the following by completing the square. Give your answers as surds.

a) $x^2 - 4x - 2 = 0$ b) $2x^2 + 4x - 7 = 0$

16) a) Complete the square for $x^2 + 6x + 8$.

b) Hence sketch the graph of $y = x^2 + 6x + 8$, labelling the turning point and intercepts with the x -axis.

Algebraic Fractions, Inequalities and Simultaneous Equations

17) Simplify the following:

a) $\frac{15a^3b^3}{5a^2b}$ b) $\frac{2x^2y}{(4xy)^2}$ c) $\frac{x^2 - 16}{x^2 - x - 20}$

These topics are on p.22-29.

18) Simplify the following:

a) $\frac{9b^2}{a} \times \frac{2a^2}{3b}$ b) $\frac{2(x-1)^2}{15} \times \frac{10}{4x-4}$ c) $\frac{3x^2 - 21x}{x+2} \div \frac{x(x-7)}{9x+18}$ d) $\frac{3}{x+1} + \frac{2x-3}{x^2}$

19) Solve the following inequalities:

a) $7x + 5 \leq 2x$ b) $2(10 - x) > 4$ c) $2x^2 + 3 < 21$
d) $4x^2 - 9 \geq 7$ e) $x^2 - 4x + 10 \geq 2x + 5$

20) Draw a set of axes with the x -axis from -2 to 3 and the y -axis from 0 to 6 .
Show on these axes the region that satisfies the following inequalities:

$$y > 3x - 1, \quad y < x + 3 \quad \text{and} \quad y \geq \frac{x}{5} + 2$$

21) Solve the following simultaneous equations:

a) $2x + y = 2$
 $x - 3y = 8$ b) $3x - 2y = 1$
 $5x - 3y = 7$ c) $y = x^2 + 3$
 $y - 2x = 18$ d) $3y = 2(x^2 - 3)$
 $2x - y = 2$

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Proof and Functions

These topics are covered on p.30-33.

- 22) Prove that the sum of any three consecutive odd numbers is a multiple of 3.
- 23) Mia says, "for any integers x and y , $xy > y$ ". Prove that Mia is wrong.
- 24) $f(x) = \frac{x+5}{3}$ and $g(x) = x - 3$.
- a) Evaluate $f(4)$. b) Find $fg(x)$. c) Find $f^{-1}(x)$.

Straight Lines and Quadratic Graphs

- 25) Give the gradient and y -intercept of the line $x + 2y = 4$.
- 26) Point A has coordinates (5, 2) and point B has coordinates (2, -4).
- a) Find the equation of the line passing through points A and B.
- b) Find the exact length of line AB.
- 27) Line A has equation $y = 2x + 5$.
- a) Find the equation of the line parallel to line A which passes through (3, 2).
- b) Find the equation of the line perpendicular to line A which passes through (2, 1).
- 28) Sketch the graph of $y = x^2 - 8x + 15$. Label the graph with the coordinates of the turning point and the points where the graph crosses the axes.

Go to p.34-37 if you found these questions tricky.

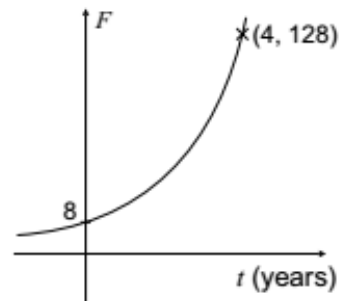
Harder Graphs and Graph Transformations

You can brush up on the skills needed for these questions on p.38-41.

- 29) Sketch the following graphs:

a) $y = x^3$ b) $y = \frac{1}{x}$ c) $y = -\frac{1}{x}$

- 30) The graph on the right shows how the number of fish (F) living in a river changes over time. The equation of the graph is $F = mn^t$ where t is the number of years and m and n are positive constants. Find the values of m and n .



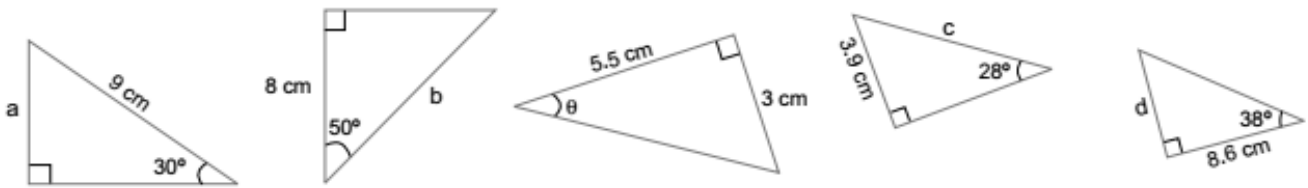
- 31) Find the equation of the tangent to $x^2 + y^2 = 25$ at the point (3, 4). Give your answer in the form $ax + by + c = 0$.
- 32) $f(x) = x^2$. For parts a) to c) below, sketch the graphs of $y = f(x)$ and the given transformation.
- a) $y = f(x) + 3$ b) $y = f(x + 3)$ c) $y = -f(x)$

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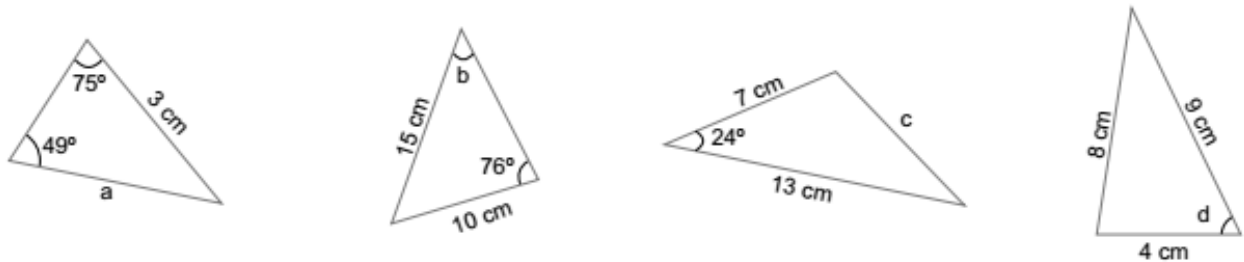
Trigonometry and Vectors

These topics are in Section 6 — p.42-50.

33) Find the unknowns in each of these triangles. Give your answers to 1 decimal place.



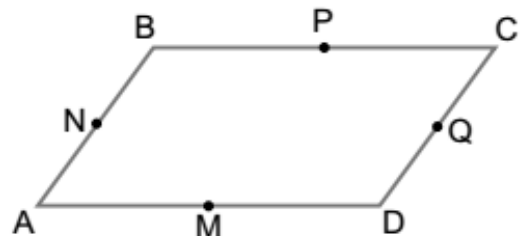
34) Find the unknowns in each of these triangles. Give your answers to 1 decimal place.



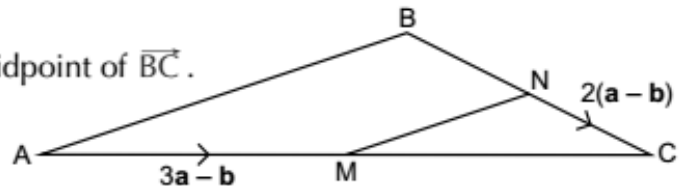
35) ABCD is the parallelogram shown on the right. M, N, P and Q are the midpoints of the sides. $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$.

Find the following vectors in terms of \mathbf{a} and \mathbf{b} .

- a) \overrightarrow{AC} b) \overrightarrow{DQ} c) \overrightarrow{CM}
 d) \overrightarrow{QP} e) \overrightarrow{MB} f) \overrightarrow{PA}



36) The diagram shows triangle ABC. M is the midpoint of \overline{AC} and N is the midpoint of \overline{BC} . $\overrightarrow{AM} = 3\mathbf{a} - \mathbf{b}$ and $\overrightarrow{NC} = 2(\mathbf{a} - \mathbf{b})$. Show that \overline{AB} and \overline{MN} are parallel.



Sampling and Histograms

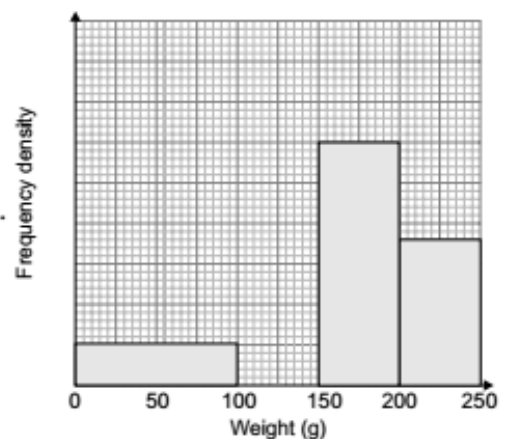
You'll find these topics on p.51-53.

37) Describe how a simple random sample of size 20 can be selected from a population of 200.

38) The weights of the chocolate bars in a shop storeroom are shown in the table and histogram below.

- a) Use the information in the table and the histogram to label the vertical axis.
 b) Use the histogram to complete the table.
 c) Use the table to add the missing bar to the histogram.

Weight (w , in grams)	Frequency
$0 < w \leq 100$	50
$100 < w \leq 150$	100
$150 < w \leq 200$	150
$200 < w \leq 250$	



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Averages and Cumulative Frequency

Averages are covered on p.54-55.

39) Find the mean, median and mode(s) of these numbers:

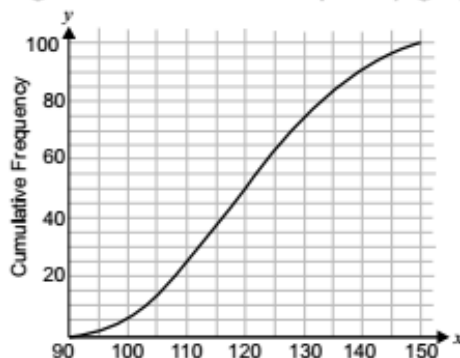
5 3 -2 0 -3 2 1 1 4 2 6 11 -4

40) The table shows the journey times between home and school for 60 students.

- Write down the modal class.
- Which group contains the median?
- Estimate the mean value.
- Draw a cumulative frequency graph for the data in the table.

Time (m minutes)	Frequency
$5 < m \leq 10$	4
$10 < m \leq 15$	25
$15 < m \leq 20$	18
$20 < m \leq 25$	8
$25 < m \leq 30$	5

41) Using this cumulative frequency graph, find the:



- median
- lower quartile
- upper quartile
- interquartile range

You can learn about cumulative frequency on p.56.

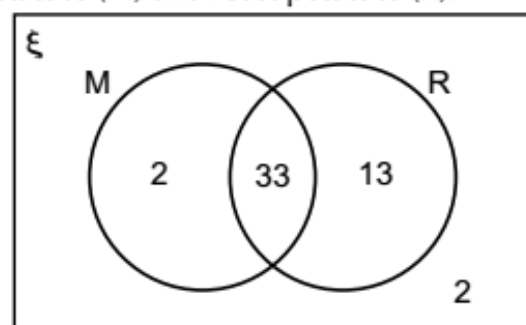
Probability and Tree Diagrams

You can learn about these topics on p.57-60.

42) Lewis asked 50 people if they like mashed potatoes (M) and roast potatoes (R). The Venn diagram shows the results.

A person is chosen at random.
Find the probability that they:

- like mashed potatoes
- like neither mashed nor roast potatoes
- like both types of potatoes
- don't like roast potatoes
- don't like mashed potatoes



43) Mona's purse contains two £5 notes, four £10 notes and three £20 notes. It also contains five 20p coins, four 50p coins and three £1 coins.

- Mona picks one note and one coin at random from her purse. Find the probability that she picks a £5 note and a 20p coin.
- Mona picks two coins at random without replacement. Use a tree diagram to find the probability she picks a 50p coin and a £1 coin.